Lecture 18

AoI and Sampling

Reading: Wait or Update TIT 2017.

JSAC AoI survey

Sun, Cyr 2019.

Ornee, Sun 2020

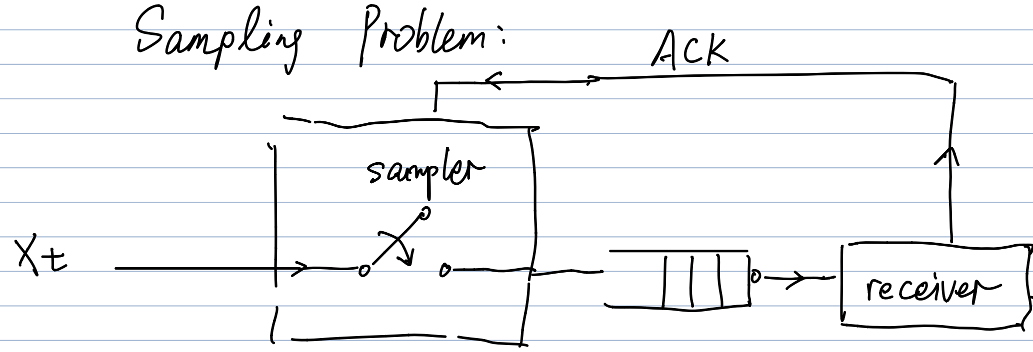
- When the arrival process is fixed,

  LGFS is (near) optimal for scheduling.

- When the arrival process is controllable.

  How to pick the generation & arrival time?

Sampling Problem:



ACK: zero feedback delay

server idle/busy state is known at the sampler.

Sample i: generation time S_i delivery time D_i.

$$\Delta(t) = t - \max\{S_i : D_i \le t\}.$$

Consider a FCFS queueing system, with general service time distribution.

Y_i: service time of sample i.

$Y_i \sim$ i.i.d.

- If a sample is generated when server is busy, the packet has to wait in the queue for transmission, and becomes stale while waiting.
- it is better not to generated samples when the channel is busy. That is, samples are taken only when the server is idle.
- Nature strategy: just-in-time updating. submit a fresh packet once the previous packet is delivered and an ACK is received.

  —— also called zero-wait policy.
- zero-wait is throughput optimal.

  —— server is always busy.

  $$T = \frac{1}{E[Y_i]}.$$
- zero-wait is delay optimal.

  —— waiting time in the queue = 0,

  delay = mean service time = $E[Y_i]$.

  minimum possible delay.

- Surprise: Zero-wait is not age-optimal.

Example: service time $Y_i = 0$ or $2s$ with prob. 0.5.

Suppose that Sample 1 is taken at time $t=0$.

with a zero service time $Y_1 = 0$.

Question: When to take Sample 2?

- zero-wait:
  - server ↘ idle at time $t=0$.

    take Sample 2 at $t=0$.
  - Samples 1 & 2 are both taken at time $t=0$.

    After Sample 1 is delivered, Sample 2 cannot bring new information to the receiver.
  - Sample 2 occupies the channel busy 1 second on avg.

    No gain, only pain.

    Zero wait may not be the best choice.
- ε-wait:

  Wait for ε seconds, if Y_i of previous sample $=0$.

  wait for 0 s, if Y_i ---- $= 2s$.

average age of ε- wait:

$$\bar{\Delta} = (\varepsilon^2 + 2\varepsilon + 8) / (4 + 2\varepsilon).$$

Reading: wait or update, TIT 2017.

if $\varepsilon = 0$. zero- wait.

$$\bar{\Delta} = 2s.$$

if $\varepsilon = 0.5$ 0.5 – wait.

$$\bar{\Delta} = 1.85s.$$

zero- wait is not age-optimal!

Research goal:

Find the optimal sampling strategy minimizing AoI.

- Non-linear aging metric:

Def: $P(t)$ is a non-decreasing function

$P(\Delta(t))$.

Def: $U(t)$ is a non-increasing function.

$$U(t) = -P(t).$$

Examples:

(i) Auto-Correlation function:

$$R(\Delta(t)) = \left| E[X_t^* X_{(t-\Delta(t))}] \right|$$

For stationary sources,

$R(\Delta(t))$ is a function of $\Delta(t)$.

(ii). Real-time estimation error:

Consider a stationary Markov source X_t.

use old samples of the source to estimate the current signal value X_t.

$$W^t = \{(X_{S_i}, S_i) : D_i \le t\}.$$

$$\hat{X}_t = f(W^t).$$

$$mse_f = E\left[(X_t - \hat{X}_t)^2\right]$$

$$= E\left[(X_t - f(W^t))^2\right],$$

$$mse_{opt} = \min_f \ mse_f$$

If (1) the sampling times are indepedent of X_t, (2) X_t is a stationary Markov source, then the estimation error mse_{opt} is an non-decreasing age function.

(iii) Information Theoretic Freshess metric:

$$I(X_t; W^t) = H(X_t) - H(X_t \mid W^t).$$

is the amount of information W^t carry about the current signal value X_t.

If $I(X_t; W^t) \approx H(X_t)$, W^t is fresh.

If $I(X_t; W^t) \approx 0$, W^t is stale.

(iv) . . . .

Reading:

Section II-D.

JSAC AoI survey.

$p(t)$ is non-decreasing.

$$\bar{p}_{opt} = \inf_{\pi \in \Pi} \limsup_{T \to \infty} \frac{1}{T} E\left[\int_0^T p(\Delta(t))\, dt \right].$$

π: $(S_1, S_2, \cdots)$ is a sampling policy.

S_2 → sampling time.

Π: the set of causal sampling policies.